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On the absence of arbitrage in diffusion markets with reflection and skewness

Alexis Anagnostakis * [1], David Criens [2], Mikhail Urusov [3]

[1] Université Grenoble-Alpes – France
[2] University of Freiburg – Germany
[3] University of Duisburg-Essen – Germany

We are interested in the absence of arbitrage for single asset financial market models whose asset price process is modeled by a one-dimensional general regular diffusion (captured via scale function and speed measure). In recent work, Criens and Urusov proved precise characterizations of NA, NUPBR and NFLVR in terms of scale and speed. In particular, it was shown that these notions are violated in the presence of skewness effects or reflecting boundaries (that reflection entails such arbitrage opportunities is rather intuitive). It remained open whether weaker notions of no arbitrage can hold in the presence of skewness or reflection. The literature suggests that this is not the case. Indeed, Rossello (Insur. Math. Econ., 2012) had observed that the weaker "no increasing profit" (NIP) condition fails for an exponential skew Brownian motion model, and Buckner, Dowd and Hulley (Finance & Stochastics, 2024) showed that increasing profits exist in a reflected geometric Brownian motion model. In this talk, we explain the surprising observation that there are diffusion markets that satisfy NIP in the presence of skewness effects and reflecting boundaries. This talk is based on joint work with Alexis Anagnostakis and Mikhail Urusov.

*Speaker

Can Nash inform capital requirements? Allocating systemic risk measures

Çagin Ararat * [1]

[1] Bilkent University [Ankara] – Turkey

Systemic risk measures aggregate the risks from multiple financial institutions to find system-wide capital requirements. Though much attention has been given to assessing the level of systemic risk, less has been given to allocating that risk to the constituent institutions. We propose a Nash allocation rule that is inspired by game theory. Intuitively, to construct these capital allocations, the banks compete in a game to reduce their own capital requirements while, simultaneously, maintaining system-level acceptability. Existence and uniqueness for these Nash allocations are provided under different systemic risk settings. We demonstrate the efficacy of these Nash allocations with numerical case studies. Joint work with Zachary Feinstein.

*Speaker

Numerical solution of Lévy-driven SDEs with steep potentials and applications to simulation of electricity prices

Olga Aryasova * [1,2,3], Alexei Kulik [4], Ilya Pavlyukevich [2]

[1] Institute of Geophysics, NAS of Ukraine – Ukraine
[2] Friedrich Schiller University of Jena – Germany
[3] Igor Sikorsky Kyiv Polytechnic Institute – Ukraine
[4] Politechnika Wroclawska – Poland

Electricity prices are characterized by fluctuations around a "normal" price level, extreme upward price moves and rapid returns. One of the adequate models is based on the stochastic differential equation (SDE)

$$dX_t = -\nabla U(X_t)dt + dL_t,\ t \geq 0, \tag{1}$$

where U is a potential function rapidly increasing to $+\infty$ and L is a heavy-tailed Lévy noise. An analytic study of eq. (1) is not easy, and currently, closed form solutions are known only in a few special cases, while numerical analysis appears more promising, for example, by means of the classical Euler–Maruyama numerical scheme. However, the superlinear growth of the drift $-\nabla U$ at infinity leads to the explosion of Euler scheme approximations in finite time and produces the "NaN" output. This behavior is not specific to Lévy noise and is well known for SDEs driven by a Brownian motion. In this talk we propose a novel explicit numerical approximation scheme for the solution of (1) that does not explode and preserves all moments of the solution.

*Speaker

Approximation of optimal dividend strategies in the Cramér-Lundberg model with capital injection and self-exciting claims arrival via policy gradient methods

Paulin Aubert * [1], Etienne Chevalier , Vathana Ly Vath [2]

[1] Université d'Évry-Val-d'Essonne, Université Paris-Saclay – France
[2] Ecole Nationale Supérieure d'Informatique pour l'Industrie et l'Entreprise, Université Paris-Saclay – France

In this work, we focus on dividend payment optimization problems. This field was inspired by ruin theory, and we owe the first results to de Finetti. In the literature, the case where the cash reserve follows a Cramér-Lundberg model and the counting process is a Poisson process has been widely studied, and some optimal solutions have been known since the 1970s. Since then, numerous extensions have been studied, including reinsurance and capital injections. At the same time, recent advances in the field of Hawkes processes have strengthened insurance companies' interest in these processes because, thanks to their self-excitation property, they more accurately represent the occurrence of claims in insurers' portfolios. This is why we are interested in an extension of the classical model in which the counting process is a Hawkes process and shareholders can inject capital. The addition of the Hawkes process involves solving a two-dimensional stochastic control problem, which makes it difficult to find analytical solutions. Thus, in this work, a theoretical characterization of the solution is produced and used to develop a numerical method for approximating optimal strategies. The proposed numerical method makes use of recent advances in reinforcement learning, and we illustrate its effectiveness by first comparing theoretical results for the classical case (compound Poisson) with those obtained by our numerical method. This work is a joint work with Etienne Chevalier and Vathana Ly Vath.

*Speaker

Common Noise by Random Measures: Mean-Field Equilibria for Competitive Investment and Hedging

Dirk Becherer * [1], Stefanie Hesse [1]

[1] Humboldt University of Berlin – Germany

(joined work with Stefanie Hesse, https://doi.org/10.48550/arXiv.2408.01175) We study mean-field games where common noise dynamics are described by integer-valued random measures, for instance Poisson random measures, in addition to Brownian motions. In such a framework, we describe Nash equilibria for mean-field portfolio games of both optimal investment and hedging under relative performance concerns with respect to exponential (CARA) utility preferences. Agents have independent individual risk aversions, competition weights and initial capital endowments, whereas their liabilities are described by contingent claims which can depend on both common and idiosyncratic risk factors. Liabilities may incorporate, e.g., compound Poisson-like jump risks and can only be hedged partially by trading in a common but incomplete financial market, in which prices of risky assets evolve as Itô-processes. Mean-field equilibria are fully characterized by solutions to suitable McKean-Vlasov forward-backward SDEs with jumps, for whose we prove existence and uniqueness of solutions, without restricting competition weights to be small. A novel change of measure argument and one-to-one relation to an auxiliary mean field game play key roles for proof, helping among other things to avoid restrictive conditions for an approach by direct fixed point contraction.

*Speaker

The role of agent's preferences in Sannikov's model

Filippo Beretta * [1], Dylan Possamaï [1]

[1] ETH Zurich – Switzerland

We study originally introduced by Sannikov, building on the rigorous analyses provided in recent studies by Possamaï and Touzi, as well as Possamaï and Rossato. The principal hires an agent to perform a task and compensate him with running payments throughout the contract's duration, which concludes at a random time, potentially with a lump-sum payment upon termination. In the setting of a risk-averse agent, this work aims to go beyond the power utility grow discussed in the aforementioned studies, presenting results general enough to tackle the most classical utility functions as exponential, negative power and logarithmic utility. As we shall see, the framework proposed by Possamaï and Touzi represents an intermediate scenario between two extreme regimes, termed Grows More than Power (GMP) and Grows Less than Power (GLP). In the former, the problem is degenerate whenever the principal is strictly more impatient than the agent, while in the latter degeneracy never occurs. Finally, we examine the case of a risk-neutral agent. This scenario, characterized by the explosive feature of the associated Hamiltonian, leads to a series of singular stochastic control problems. We provide a complete, explicit solution for this case, resulting in optimal contracts that differ substantially from those in the risk-averse framework. Notably, these contracts are characterised by a singular nature, with lump-sum payments at initiation (a 'welcome bonus') and the involvement of local times (a 'recurring bonus' once some objectives are reached) throughout.

*Speaker

Decentralized exchange liquidity based options

Rostislav Berezovskii * [1,2]

[1] Vysšaja škola èkonomiki = National Research University Higher School of Economics [Moscow] – Russia
[2] Vega Institute Foundation – Russia

The talk describes a new type of financial instrument existing in the Decentralized Finance setting: an onchain option. A short intro for the decentralized exchanges (DEX) and lending markets theory will be provided. The research includes a detailed design and a pricing approach for the novel derivative considering two assumptions for accumulated DEX fee estimation: constant fee rate and time dependent fee rate.

*Speaker

Discrete approximation of risk-based prices under volatility uncertainty

Jonas Blessing * [1], Michael Kupper [2], Alessandro Sgarabottolo [3]

[1] ETH Zurich – Switzerland
[2] University of Konstanz – Germany
[3] Bielefeld University – Germany

We discuss the asymptotic behaviour of risk-based indifference prices of European contingent claims in discrete-time financial markets under volatility uncertainty as the number of intermediate trading periods tends to infinity. The asymptotic risk-based prices form a strongly continuous convex monotone semigroup which is uniquely determined by its infinitesimal generator and therefore only depends on the covariance of the random factors but not on the particular choice of the model. We further compare the risk-based price with the worst-case prices given by the G-expectation and investigate their asymptotic behaviour as the risk aversion of the agent tends to infinity. The theoretical results are illustrated with several examples and numerical simulations showing, in particular, that the risk-based prices lead to a significant reduction of the bid-ask spread compared to the worst-case prices.

*Speaker

Fractional Wiener Chaos and what may or may not be a stochastic integral of fractional order

Elena Boguslavskaya * [1]

[1] Brunel University of London – United Kingdom

First we introduce a fractional analogue of the Wiener polynomial chaos expansion. It is important to highlight that the fractional order relates to the order of chaos decomposition elements, and not to the process itself, which remains the standard Wiener process. The central instrument in our fractional analogue of the Wiener chaos expansion is a function which we call a power-normalised parabolic cylinder function. Through analysis of several fundamental deterministic and stochastic properties, we affirm that this function essentially serves as a fractional extension of the Hermite polynomial on positive real line. In particular, the power-normalised parabolic cylinder function with the Wiener process and time as its arguments demonstrates martingale properties and can be interpreted as a fractional Ito integral with 1 as the integrand, thereby drawing parallels with its non-fractional counterpart. To build a fractional analogue of polynomial Wiener chaos on the whole real line, we introduce a new function, which we call the extended Hermite function, by smoothly joining two power-normalized parabolic cylinder functions at zero. We form an orthogonal set of extended Hermite functions as a one-parameter family and use tensor products of the extended Hermite functions as building blocks in the fractional Wiener chaos expansion, in the same way that tensor products of Hermite polynomials are used as building blocks in the Wiener chaos polynomial expansion. Finally, we discuss the differences between the classical and fractional Wiener Chaos, and explore the ways a stochastic integral of fractional order may be defined.

The talk is based on the joint work with Elina Shishkina.

*Speaker

Finite-time horizon, stopper vs. singular-controller games on the half-line.

Andrea Bovo * [1], Tiziano De Angelis [1,2]

[1] University of Turin – Italy
[2] Collegio Carlo Alberto – Italy

We prove existence of a value for two-player zero-sum stopper vs. singular-controller games on finite-time horizon, when the underlying dynamics is one-dimensional, diffusive and bound to evolve in the positive real line. We show that the value is the maximal solution of a variational inequality with both obstacle and gradient constraint and satisfying a Dirichlet boundary condition at 0. Moreover, we obtain an optimal strategy for the stopper. In order to achieve our goals, we rely on new probabilistic methods, yielding gradient bounds and equi-continuity for the solutions of penalised partial differential equations that approximate the variational inequality.

*Speaker

Adversarial Schrödinger Bridge Matching for Fast Generative Modeling

Evgeny Burnaev * [1]

[1] Skolkovo Institute of Science and Technology [Moscow] – Russia

The Schrödinger Bridge (SB) problem offers a powerful framework for combining optimal transport and diffusion models. A promising recent approach to solve the SB problem is the Iterative Markovian Fitting (IMF) procedure, which alternates between Markovian and reciprocal projections of continuous-time stochastic processes. However, the model built by the IMF procedure has a long inference time due to using many steps of numerical solvers for stochastic differential equations. To address this limitation, we propose a novel Discrete-time IMF (D-IMF) procedure in which learning of stochastic processes is replaced by learning just a few transition probabilities in discrete time. Its great advantage is that in practice it can be naturally implemented using the Denoising Diffusion GAN (DD-GAN), an already well-established adversarial generative modeling technique. We show that our D-IMF procedure can provide the same quality of unpaired domain translation as the IMF, using only several generation steps instead of hundreds.

*Speaker

No arbitrage in the quasi-sure projective setting

Laurence Carassus * [1]

[1] CentraleSupélec, Université Paris-Saclay – France

In the context of non-dominated model uncertainty, the use of projective sets instead of analytic sets has been fruitful for non-concave utility maximisation. Assuming the set-theorical axiom of Projective Determinacy, projective sets share the same properties as analytic sets. Recall that in the classic setup of Bouchard and Nutz, prices are Borelian, graphs of the random sets of beliefs analytically measurable and strategies universally measurable. The projective setting unifies the measurability of the various objects in the model. Indeed assuming that the prices and the graph of the random sets of beliefs are projectively measurable, existence of projectively measurable solutions to the nonconcave utility maximization problem has been proved. The aim of this paper is to propose a characterisation of the quasi-sure no-arbitrage assumption in this projective setup, where prices and the beliefs graph are assumed to be projectively measurable and the Projective Determinacy axiom postulated.

*Speaker

Sharp results for monotone mean-variance portfolios with general independent increments

Aleš Černý * [1], Johannes Ruf [2], Martin Schweizer [3]

[1] Bayes Business School, City St George's, University of London – United Kingdom
[2] LSE Mathematics – United Kingdom
[3] ETH Zurich – Switzerland

Assuming less than the existence of an equivalent martingale measure and placing no requirements on the integrability of the returns process, we provide a complete solution of the optimal monotone mean-variance portfolio allocation for returns with semimartingale independent increments. Illustrative examples are given.

*Speaker

A short note on super-hedging an arbitrary number of European options with integer-valued strategies

Dorsaf Cherif * [1], Emmanuel Lepinette , Meriem El Mansour

[1] Carthage University – Tunisia

The usual theory of asset pricing in finance assumes that the financial strategies, i.e. the quantity of risky assets to invest, are real-valued so that they are not integer-valued in general, see the Black and Scholes model for instance. This is clearly contrary to what it is possible to do in the real world. Surprisingly, it seems that there are not many contributions in that direction in the literature, except for a finite number of states. In this paper, for arbitrary Ω, we show that, in discrete-time, it is possible to evaluate the minimal super-hedging price when we restrict ourselves to integer-valued strategies. To do so, we only consider terminal claims that are continuous piecewise affine functions of the underlying asset. We formulate a dynamic programming principle that can be directly implemented on historical data and which also provides the optimal integer-valued strategy. The problem with general payoffs remains open but should be solved with the same approach.

*Speaker

Super-hedging-pricing formulas for vulnerable claims in market models with random horizon and its induced arbitrage

Tahir Choulli * [1]

[1] University of Alberta – Canada

We consider the discrete-time setting, and the market model described by (S, F,T). Herein F is the "public" flow of information which is available to all agents overtime, S is the discounted price process of d-tradable assets, and T is an arbitrary random time whose occurrence might not be observable via F. Thus, we consider the larger flow G which incorporates F and makes T an observable random time. This framework covers the credit risk theory setting where T is the default time of a firm or a client, the life insurance setting where T is the death time of an insured, and the setting of employee stock option valuation. For the stopped model $(Sˆ T,G)$ and for various vulnerable claims, based on this model, we address the super-hedging pricing valuation problem and its intrinsic Immediate-Profit arbitrage (IP hereafter for short). Our first main contribution lies in singling out the impact of change of prior and/or information on conditional essential supremum, which is a vital tool in super-hedging pricing. The second main contribution consists of describing as explicit as possible how the set of super-hedging prices expands under the stochasticity of T and its risks, and we explain how the IP arbitrage for $(Sˆ T,G)$ is triggered. The third main contribution resides in elaborating an explicit and backward pricing formulas for vulnerable claims, and singling out the various informational risks in the prices' dynamics. This talk is based on joint work with Emmanuel Lepinette (Paris-Dauphine, France)

*Speaker

Optimal control under unknown intensity with Bayesian learning

Quentin Cormier * [1], Nicolas Baradel

[1] Inria Saclay, Institut Polytechnique de Paris – France

We consider an optimal control problem inspired by neuroscience, where the dynamics is driven by a Poisson process with a controlled stochastic intensity and an uncertain parameter. Given a prior distribution for the unknown parameter, we describe its evolution according to Bayes' rule. We reformulate the optimization problem using Girsanov's theorem and establish a dynamic programming principle. Finally, we characterize the value function as the unique viscosity solution to a finite-dimensional Hamilton-Jacobi-Bellman equation, which can be solved numerically.

*Speaker

Stochastic Processes under Uncertainty

David Criens * [1]

[1] University of Freiburg – Germany

In mathematical finance asset prices are often modeled as parametric stochastic processes. For example, the famous Black-Scholes model assumes that prices follow geometric Brownian motions with certain drift and volatility parameters. In practice, the parameters cannot be identified precisely, which leads to parameter uncertainty within the model class. In this lecture I will explain several modern approaches to define and analyize stochastic processes under parameter uncertainty. Taking a closer look at Peng's G-Brownian motion, we learn about semigroup and control approaches and discuss the important PDE connection. I will also discuss extensions to the class of G-diffusions and some state of the art techniques to establish Feller properties of sublinear semigroups, which correspond to a priori continuity of value functions in the language of optimal control. Further, I will comment on financial applications. The final part of the lecture takes a short look at the discrete time model of Bouchard and Nutz with a spotlight on the relation to continuous time models.

*Speaker

Statistical Learning of Value-at-Risk and Expected Shortfall

Stéphane Crépey * [1]

[1] Université Paris Cité – France

We propose a non-asymptotic convergence analysis of a two-step approach to learn a conditional value-at-risk (VaR) and a conditional expected shortfall (ES) using Rademacher bounds, in a non-parametric setup allowing for heavy-tails on the financial loss. Our approach for the VaR is extended to the problem of learning at once multiple VaRs corresponding to different quantile levels. This results in efficient learning schemes based on neural network quantile and least-squares regressions. An a posteriori Monte Carlo procedure is introduced to estimate distances to the ground-truth VaR and ES. This is illustrated by numerical experiments in a Student-t toy model and a financial case study where the objective is to learn a dynamic initial margin. Joint work with David Barrera, Emmanuel Gobet, Hoang-Dung Nguyen, and Bouazza Saadeddine.

*Speaker

Signature methods in finance

Christa Cuchiero * [1]

[1] University of Vienna – Austria

Signature methods represent a non-parametric way for extracting characteristic features from time series data which is essential in machine learning tasks, mathematical finance and risk assessment. Indeed, signature based approaches allow for data-driven and thus more robust model selection mechanisms, while first principles like no arbitrage can still be easily guaranteed. One focus of this talk lies on the use of signature as universal linear regression basis of certain continuous paths functionals in financial applications. In these applications key quantities that have to be computed efficiently are the expected signature or the characteristic function of the signature of some underlying stochastic process. Surprisingly this can be achieved for generic classes of diffusion processes, called signature SDEs, via techniques from affine and polynomial processes. In terms of concrete applications we present several recent contributions ranging from signature based asset price models for joint VIX and SPX calibration, over control problems in stochastic portfolio theory to functional Taylor expansions of path-dependent options.

The talk is based on several joint works with Guido Gazzani, Xin Guo, Janka Möller, Francesca Primavera, Sara-Svaluto Ferro and Josef Teichmann.

*Speaker

Mean-field control of non exchangeable systems

Anna De Crescenzo * [1], Marco Fuhrman [2], Idris Kharroubi [3], Huyen Pham [4]

[1] Université Paris Cité – France
[2] Università degli studi di Milano – Italy
[3] Sorbonne Universités – France
[4] Ecole Polytechnique, Institut Polytechnique de Paris – France

We study the optimal control of mean-field systems with heterogeneous and asymmetric interactions. This leads to considering a family of controlled Brownian diffusion processes with dynamics depending on the whole collection of marginal probability laws. We prove the well-posedness of such systems and define the control problem together with its related value function. We next prove a law invariance property for the value function which allows us to work on the set of collections of probability laws. We show that the value function satisfies a dynamic programming principle (DPP) on the flow of collections of probability measures. We also derive a chain rule for a class of regular functions along the flows of collections of marginal laws of diffusion processes. Combining the DPP and the chain rule, we prove that the value function is a viscosity solution of a Bellman dynamic programming equation in a Lˆ2-set of Wasserstein space-valued functions.

*Speaker

Energy efficiency and demand response: a mean-field game approach.

Roxana Dumitrescu * [1]

[1] ENSAE, Institut Polytechnique de Paris – France

In this talk, I will present two models developed in the context of energy transition, using as mathematical tools the theory of mean-field games and the principal agent-mean-field game approach. The first model is related to demand response, in which we consider an energy system with a large number of consumers who are linked by a Demand Side Management contract, i.e. they agree to diminish, at random times, their aggregated power consumption by a predefined volume. We provide numerical results which illustrate the impact of such an interaction on the consumption and price levels. The numerical results are obtained by developing new deep learning solvers for coupled FBSDEs. The second model focuses on the problem of an energy retailer aiming at designing a new type of contract based on a ranking system for a population of heterogeneous consumers to incentivise them to make energy economies (based on several joint works with C. Alasseur, E. Bayraktar, L. Campi , Q. Jacquet, X. Warin , J. Zeng)

*Speaker

Optimal portfolio choice in complete jump-diffusion markets with longevity risk

Davide Feleppa * [1], Immacolata Oliva [1]

[1] Department of Methods and Models for Territory, Economics and Finance, Sapienza University of Rome, Rome – Italy

In the present paper, we provide optimal portfolio choice for an investment strategy endowed with Target-date funds. The proposal must ensure a minimum level of gain from the investment at maturity, and hypothesizes uncertainty in interest rate, contribution rate, and mortality. Moreover, the financial setting assumes the presence of discontinuities in the risky asset dynamics to reflect the occurrence of market crashes. To hedge against investment, longevity and event risks, we complete the market by using a zero-coupon bond, a longevity zero-coupon bond, and a derivative. We apply standard dynamic programming techniques and obtain closed-form solutions to the stochastic control problem with the objective of maximizing the expected utility of terminal wealth. We complete the picture by performing an extensive numerical analysis on real data, to measure the impact of market crashes and the effect of hedging tools.

*Speaker

Theoretical guarantees in KL for Diffusion Flow Matching

Marta Gentiloni Silveri * [1]

[1] Ecole Polytechnique, Institut Polytechnique de Paris – France

Flow Matching (FM) (also referred to as stochastic interpolants or rectified flows) stands out as a class of generative models that aims to bridge in finite time the target distribution $\nu^\star$ with an auxiliary distribution μ leveraging a fixed coupling π and a bridge which can either be deterministic or stochastic. These two ingredients define a path measure which can then be approximated by learning the drift of its Markovian projection. The main aim of this talk is to provide relatively mild assumption on $\nu^\star$, μ and π to obtain non-asymptotics guarantees for Diffusion Flow Matching (DFM) models using as bridge the conditional distribution associated with the Brownian motion. More precisely, I will present bounds on the Kullback-Leibler divergence between the target distribution and the one generated by such DFM models that hold true under moment conditions on the score of $\nu^\star$, μ and π, and a standard L^2-drift-approximation error assumption.

*Speaker

Chain or Channel? Payment Optimization with Heterogeneous Flow

Paolo Guasoni * [1]

[1] School of Mathematical Sciences, Dublin City University – Ireland

Payment-channel networks (PCNs) such as the Lightning Network enable offchain payments secured by the channels' balances as alternatives to on-chain transactions. This paper solves the optimal channel management problem for two agents who pay each other arbitrarily distributed amounts. Agents optimally choose the channel's size and whether to make each payment on-chain or on-channel, depending on their current balance. With unidirectional flows, payments below some balancedependent chain amount happen on-channel while others on-chain. As the balance falls below the reserve level, payments are always made on-channel if feasible. Below the refill level, the channel is reset to its initial state. Symmetric bidirectional flows entail distinct chain thresholds and reset levels for both directions, but channels may last indefinitely. Asymmetric flows lead to a more complex optimal policies, in which both, either, or no party resets the channel. The paper characterizes optimal channels and payment policies, describing an algorithm to obtain them, given payments' frequency and distribution.

*Speaker

The optimal switching problem with signed switching costs

Said Hamadene * [1]

[1] Le Mans Université – France

In this talk we discuss the optimal multiple modes switching problem in finite horizon when the costs associated with the changes of regimes do not have a constant sign. The problem is solved by means of probabilistic tools. The main assumption is the monotonicity of the switching costs. In the Markov setting, the associated HJB system of PDEs is also considered. We show the existence and uniqueness of the solution in viscosity sense. Switching problems get involved in energy markets, financial markets, cyber-security field, etc. This is a joint work with B.ElAsri and M.Souheil (Agadir University).

*Speaker

No Tick-Size Too Small: A General Method for Modelling Small Tick Limit Order Books

Konark Jain * [1,2], Jean-François Muzy [3], Jonathan Kochems [2], Emmanuel Bacry [4]

[1] Department of Computer science, University College of London – United Kingdom
[2] JP Morgan Chase & Co. – United Kingdom
[3] Universitede Corsica – France
[4] Université Paris-Dauphine, PSL – France

We investigate the disparity in the microstructural properties of the Limit Order Book (LOB) across different relative tick sizes. Tick sizes not only influence the granularity of the price formation process but also affect market agents' behavior. A key contribution of this study is the identification of several stylized facts, which are used to differentiate between large, medium, and small tick stocks, along with clear metrics for their measurement. We provide cross-asset visualizations to illustrate how these attributes vary with relative tick size. Further, we propose a Hawkes Process model that accounts for sparsity, multi-tick level price moves, and the shape of the book in small-tick stocks. Through simulation studies, we demonstrate the universality of the model and identify key variables that determine whether a simulated LOB resembles a large-tick or small-tick stock. Our tests show that stylized facts like sparsity, shape, and relative returns distribution can be smoothly transitioned from a large-tick to a small-tick asset using our model. We test this model's assumptions, showcase its challenges and propose questions for further directions in this area of research.

*Speaker

A Forward-Backward Approach to Endogenous Distress Contagion

Philipp Jettkant * [1], Andreas Søjmark [2]

[1] Imperial College London – United Kingdom
[2] London School of Economics and Political Sciences – United Kingdom

In this talk, I will introduce a dynamic model of a banking network in which the value of interbank obligations is continuously adjusted to reflect counterparty default risk. An interesting feature of the model is that the credit value adjustments increase volatility in times of distress, leading to endogenous default contagion between the banks. The counterpart default risk can be computed backwards in time from the maturity date, leading to a specification of the model in terms of a forward-backward stochastic differential equation (FBSDE), coupled through the banks' default times. While one can prove the existence of solutions with minimal and maximal default probabilities, the question of uniqueness is currently open. I will conclude the talk by discussing a characterisation of the maximal default probabilities through a cascade of partial differential equations (PDEs), each representing a configuration with a different number of defaulted banks. The domain of each PDE has a free boundary that coincides with the banks' default thresholds.

*Speaker

On the 90th Anniversary of Albert Shiryaev

Youri Kabanov * [1]

[1] Université de Franche-Comté [Besançon] – France

*Speaker

Propagation of chaos for a particle system related to the mean-field Schrödinger dynamics

Anna Kazeykina * [1], Jiacheng Zhang [2]

[1] Université Paris Saclay – France
[2] Chinese University of Hong Kong – China

The mean-filed Scrodinger (MFS) dynamics arises naturally in the context of optimisation problems regularised via Fischer information. We consider a particle system that can be used to simulate empirically the solution of MFS. We prove a finite-time propagation of chaos result in the metric closely related to Wasserstein distance and the total variation distance. Finally, we illustrate our theoretical findings by numerical simulations.

*Speaker

Super CARMA Processes for Stochastic Modeling of Renewable Energy Variables

Asma Khedher * [1], Sven Karbach , Fred Espen Benth

[1] Univeristy of Amsterdam – Netherlands

We introduce a novel class of measure-valued stochastic processes, called super continuous-time autoregressive moving-average (super CARMA) processes, and explore their applications in modeling the dynamics of functionals of spatio-temporal stochastic fields. A super CARMA process is defined as the analytically weak solution of a linear state-space model in the Banach space of finite signed measures, driven by a Lévy subordinator. By specifying particular input, transition and output operators of the linear state-space model, we demonstrate that the solution possesses CARMA dynamics and takes values in the cone of positive finite measures on a given spatial domain. This framework connects naturally to existing CARMA-type models in the literature. Additionally, we propose the class of super CARMA processes as a versatile and analytically tractable tool for capturing (i) the dynamics of flow forwards in energy markets, and (ii) the evolution of renewable energy production and consumption across geographic regions. We further show that these models allow for closed-form solutions in pricing options on power futures, providing a robust stochastic modeling tool for quantiative energy finance.

*Speaker

Optimal Spread Trading

Aleksei Kozhevnikov * [1,2]

[1] Lomonosov Moscow State University – Russia
[2] Vega Institute Foundation – Russia

This presentation explores various aspects of the optimal spread trading problem. The market is modeled with a bond and a risky asset, where the risky asset follows an Ornstein-Uhlenbeck process, representing the spread. An analytical solution to the consumption-investment problem is provided for both finite and infinite horizon settings, under the assumption of power utility. The talk is based on joint work with Yuri Kabanov.

*Speaker

Transversality Condition Matters: Ensuring Uniqueness of Solutions in Deep Learning Continuous Time Models

Aleksei Minabutdinov * [1]

[1] ETH Zurich – Switzerland

Transversality is an important sufficient condition for identifying the solution in infinite horizon economic and financial models. Without such a condition, there usually exists a continuum of functions that satisfy the Hamilton-Jacobi-Bellman (HJB) functional equation. In this paper, we explore this manifold of solutions using numerical and analytical methods. Using a standard continuous-time model, we demonstrate that, without explicitly imposing the transversality condition, widely used numerical algorithms, including the (Deep) Galerkin-type methods, are prone to converge to arbitrary points of this manifold, leading to significant and uncontrollable biases. We propose a novel approach based on a functional transformation of the original HJB equation to effectively incorporate the transversality condition, ensuring convergence to the actual value function.

*Speaker

Finite-Sample Convergence Bounds for Trust Region Policy Optimization in Mean Field Games

Antonio Ocello * [1]

[1] École Polytechnique, Institut Polytechnique de Paris – France

This presentation introduces a novel approach for solving Mean Field Games (MFGs) in finite state and action spaces, focusing on the ergodic problem. We build upon Trust Region Policy Optimization (TRPO), a Reinforcement Learning (RL) algorithm renowned for its stability and robust policy updates. Specifically, we propose an adaptation for MFGs, referred to as MF-TRPO, and provide a detailed finite-sample convergence analysis. Our results offer non-asymptotic guarantees for convergence to Nash equilibria by incorporating entropic regularization, which stabilizes policy updates and improves exploration.

The presentation begins by introducing the fundamental principles of RL and its application to MFGs. We present explicit performance bounds for MF- TRPO and examine the influence of key algorithmic parameters, such as learning rates and regularization terms, on convergence speed and sample efficiency. This work contributes to the theoretical understanding of policy-based reinforcement learning methods in large-scale MFGs and offers practical insights for their implementation in real-world scenarios.

*Speaker

Beyond the Leland strategy

Amal Omrani * [1]

[1] Université Paris-Dauphine, PSL – France

In the Black and Scholes Model with small transaction costs, Leland proposed to asymptotically super-hedge the Call option as the number of revision dates tend to $+\infty$. The idea is to use the delta-hedging strategy with an enlarged volatility to compensate for the transaction costs. The natural question is how to solve the same problem for a general model where the number of discrete dates is fixed, where the proportional transaction cost coefficient does not tends to 0 and, also, without martingale measures. We propose a new method which is easily computable.

*Speaker

Time-inhomogeneous stochastic Volterra equation

Ludger Overbeck * [1]

[1] Justus-Liebig-Universität Gießen – Germany

We extend recent results on affine Volterra processes to the inhomogeneous case. This includes moment bounds of solutions of Volterra equations driven by a Brownian motion with an inhomogeneous kernel and inhomogeneous drift and diffusion coefficients and In the case of affine drift and variance we show how the conditional Fourier-Laplace functional can be represented by a solution of an inhomogeneous Riccati-Volterra integral equation. For a time homogeneous kernel of convolution type we establish existence of a solution to the stochastic inhomogeneous Volterra equation. If in addition b and $\sigma\sigma^\top$ are affine, we prove that the conditional Fourier-Laplace functional is exponential-affine in the past path. Finally, we apply these results to an inhomogeneous extension of the rough Heston model used in mathematical finance.

Joint work with Julia Ackermann and Thomas Kruse.

*Speaker

Explosions of stochastic Volterra equations

Sergio Pulido * [1], Alessandro Bondi [2]

[1] Université d'Évry-Val-d'Essonne, ENSIIE, Université Paris-Saclay – France
[2] Ecole Polytechnique – Institut Polytechnique de Paris – France

In the first part of the presentation, we explain a Feller's test for explosions of one-dimensional continuous stochastic Volterra processes of convolution type. We focus on dynamics governed by nonsingular kernels, which preserve the semimartingale property of the processes and introduce memory features through a path-dependent drift. The results are illustrated with three specifications of the dynamics: the Volterra square-root diffusion, the Volterra Jacobi process, and the Volterra power-type diffusion. In the second part of the presentation, we provide an Osgood's test for explosions of one-dimensional stochastic Volterra equations with additive noise, featuring kernels from a family that includes the fractional kernel.

*Speaker

Propaga on of Carbon Price Shocks through the Value Chain: the Mean-Field Game of Defaults

Pavarana Simone * [1], Thorsten Schmidt [1], Peter Tankov , Zorana Grbac , Shiqi Liu

[1] Abteilung für Mathematische Stochastik, Albert-Ludwigs-Universität Freiburg – Germany

We develop a mean-field game approach to analyze the impact of carbon pricing in a multisectoral economy. To build intuition, we first introduce a simple, static, and deterministic market model in which each sector comprises a single representative firm. Assuming perfect competition, firms act as price takers, maximizing their profit by selecting an optimal mix of inputs-specifically, quantity, emissions, and labor. The carbon pricing mechanism influences each firm's cost structure by introducing a marginal cost of emissions, and sectoral prices are determined by a market-clearing condition that balances supply and demand. We establish the existence of a Nash equilibrium using the MiniMax Theorem, which allows an interpretation of our model within welfare economics and potential game theory. We then extend the model to incorporate dynamics and multiple defaultable firms per sector. Here, default is modeled as an optimal stopping problem, with firms choosing both inputs and the timing of market exit to maximize expected gains over a finite time horizon. We then take the mean-field limit, assuming an infinite number of firms per sector. Utilizing the relaxed approach by Bouveret et al., we reformulate the optimal stopping problem as an equivalent linear programming problem, with occupation measure flows and exit measures as central constructs. We prove the existence of a linear programming mean-field game Nash equilibrium and the uniqueness of the corresponding price system. Finally, we propose an algorithm to simulate and calibrate the model using real data from Exiobase 3.

*Speaker

Logarithmic regret in the ergodic Avellaneda-Stoikov market making model

Jialun Cao , David Siska * [1], Lukasz Szpruch , Tanut Treetanthiploet

[1] University of Edinburgh – United Kingdom

We analyse the regret arising from learning the price sensitivity parameter κ of liquidity takers in the ergodic version of the Avellaneda-Stoikov market making model. We show that a learning algorithm based on a regularised maximum-likelihood estimator for the parameter achieves the regret upper bound of order $(\ln T)^2$ in expectation. To obtain the result we need two key ingredients. The first are tight upper bounds on the derivative of the ergodic constant in the Hamilton-Jacobi-Bellman (HJB) equation with respect to κ. The second is the learning rate of the maximum-likelihood estimator which is obtained from concentration inequalities for Bernoulli signals. Numerical experiment confirms the convergence and the robustness of the proposed algorithm.

*Speaker

Martingale property in signature volatility models

Dimitri Sotnikov * [1], Eduardo Abi Jaber [1], Paul Gassiat [2]

[1] Ecole Polytechnique, Institut Polytechnique de Paris– France
[2]Université Paris-Dauphine, PSL, DMA ENS – France

We study a signature volatility model where the volatility process is represented as a linear form of the extended Brownian motion signature. Our objective is to identify the conditions on the coefficients of this linear form under which the spot price process is a true martingale. By generalizing classical techniques originally applicable in a Markovian setting, we prove that, in the presence of a non-zero dominant coefficient of order N in the linear form, the spot process is a true martingale if and only if N is odd and the product of the dominant coefficient and the spot-volatility correlation is non-positive. This is a joint work with Eduardo Abi Jaber and Paul Gassiat.

*Speaker

Polynomial approximation of discounted moments

Peter Spreij * [1]

[1] University of Amsterdam – Netherlands

We introduce an approximation strategy for the discounted moments of a stochastic process that can, for a large class of problems, approximate the true moments. These moments appear in pricing formulas of financial products such as bonds and credit derivatives. The approximation relies on high-order power series expansion of the infinitesimal generator, and draws parallels with the theory of polynomial processes. We demonstrate applications to bond pricing and credit derivatives. In the special cases that allow for an analytical solution the approximation error decreases to around 10 to 100 times machine precision for higher orders. When no analytical solution exists we tie out the approximation with existing numerical techniques. Joint work with Chenyu Zhao, Misha van Beek and Makhtar Ba.

*Speaker

A gradient flow on control space with rough initial condition

Florin Suciu * [1], Paul Gassiat [1]

[1] Université Paris-Dauphine, PSL – France

We consider the (sub-Riemannian type) control problem of finding a path going from an initial point x to a target point y, by only moving in certain admissible directions. We assume that the corresponding vector fields satisfy the bracket-generating (Hörmander) condition, so that the classical Chow-Rashevskii theorem guarantees the existence of such a path. One natural way to try to solve this problem is via a gradient flow on control space. However, since the corresponding dynamics may have saddle points, any convergence result must rely on suitable (e.g. random) initialisation. We consider the case when this initialisation is irregular, which is conveniently formulated via Lyons' rough path theory. We show that one advantage of this initialisation is that the saddle points are moved to infinity, while minima remain at a finite distance from the starting point. In the step 2-nilpotent case, we further manage to prove that the gradient flow converges to a solution, if the initial condition is the path of a Brownian motion (or rougher). The proof is based on combining ideas from Malliavin calculus with Lojasiewicz inequalities. A possible motivation for our study comes from the training of deep Residual Neural Nets, in the regime when the number of trainable parameters per layer is smaller than the dimension of the data vector.

*Speaker

Rough backward SDEs of Marcus-type with discontinuous Young drivers

Yuchen Sun * [1], Dirk Becherer [1]

[1] Humboldt-Universität zu Berlin

We show existence and uniqueness of backward differential equations that are jointly driven by Brownian martingales B and a deterministic discontinuous rough path W of q-variation for $1 \leq q < 2$. Integration of jumps is in the geometric sense in the spirit of Marcus-type stochastic differential equations. The well-posedness is shown through a direct fix-point argument. By developing a comparison theorem, we can derive an apriori bound of the solution, which helps us attain a unique global solution of the differential equation. Furthermore, a connection to backward doubly SDE is established. If time permits, we will further discuss the continuity of the rough backward SDE solution with respect to the terminal condition and the driving rough noise in a Skorokhod-type norm. This is a joint work with Dirk Becherer (HU Berlin).

*Speaker

Advanced Filtering Techniques for Stochastic Discontinuities

Félix Barrez Tambe Ndonfack * [1], Thorsten Schmidt [1]

[1] University of Freiburg – Germany

In this work, we develop a framework for filtering in stochastic systems with discontinuities. Addressing challenges where observations are influenced by abrupt changes, we model these phenomena using paired stochastic processes for unobserved signals and observable data streams. By incorporating Poisson random measures, our approach systematically captures the behavior of discontinuities, providing a structured method for estimating hidden states in complex systems. Key theoretical results include the derivation of the Zakai and Kushner-Stratonovich equations, which facilitate precise computation of conditional distributions, enabling refined filtering under non-continuous dynamics. This framework has significant applications in fields where sudden shifts in state variables are critical, such as financial risk assessment and advanced signal processing, offering a robust alternative to continuous filtering models.

*Speaker

Asset pricing under transition risk and model ambiguity

Peter Tankov * [1]

[1] ENSAE, Institute Polytechnique de Paris – France

How to make decisions in the presence of transition-related risks and uncertainties? One widely-used approach consists in using scenarios, to structure future outcomes and determine the consequences of policies and decisions. In this lecture, we will first review the scenario approach and then study asset pricing and optimal investment decisions for an economic agent whose revenues depend on the realization of a scenario from a given set of possible future outcomes. In the first part of the talk, we assume that future scenario is unknown, but that the possible scenarios have equal probabilities, and the agent deduces scenario information progressively by observing a signal. The problem of valuing an investment or a company is formulated as an American option pricing problem with Bayesian learning. In the second part, we assume that the probabilities of individual prospective scenarios are ambiguous and place ourselves into the smooth model of decision making under ambiguity aversion of Klibanoff et al (2005), framing the optimal investment decision as an optimal stopping problem with learning under ambiguity. We then prove a minimax result allowing to reduce this problem to a series of standard optimal stopping problems. Depending on the available time, the theory will be illustrated with several examples: optimally selling a stock with ambiguous drift, pricing a bond sensitive to transition risks, and optimal divestment from a coal-fired power plant under transition scenario ambiguity.

*Speaker

Path dependent Modeling In Finance

Josef Teichmann * [1]

[1] ETH Zurich – Switzerland

We analyse the modeling power of delay equations in Finance from the perspective of Takens' theorem in dynamical systems.

*Speaker

Machine Learning-Based Solver for Martingale Optimal Transport

Igor Udovichenko * [1,2]

[1] Skolkovo Institute of Science and Technology [Moscow] – Russia
[2] Vega Institute Foundation – Russia

This work explores the entropic regularization of the robust model-free pricing problem within the framework of martingale optimal transport, focusing on a realistic discrete-time setup with a finite number of traded instruments. We demonstrate that the dual problem can be effectively addressed through the training of a neural network. Our findings are supported by experimental results that validate the proposed methodology in this specific context.

*Speaker

Representation property for 1d general diffusion semimartingales

David Criens [1], Mikhail Urusov * [2]

[1] University of Freiburg – Germany
[2] University of Duisburg-Essen – Germany

A general diffusion semimartingale is a 1d continuous semimartingale that is also a regular strong Markov process. The class of general diffusion semimartingales is a natural generalization of the class of (weak) solutions to SDEs. A continuous semimartingale has the representation property if all local martingales w.r.t. its canonical filtration have an integral representation w.r.t. its continuous local martingale part. The representation property is closely related to market completeness. We show that the representation property holds for a general diffusion semimartingale if and only if its scale function is (locally) absolutely continuous in the interior of the state space. Surprisingly, this means that not all general diffusion semimartingales possess the representation property, which is in contrast to the SDE case. Furthermore, it follows that the laws of general diffusion semimartingales with absolutely continuous scale functions are extreme points of their semimartingale problems. We construct a general diffusion semimartingale whose law is not an extreme point of its semimartingale problem. This contributes to the solution of the problems posed by Jacod and Yor and by Stroock and Yor on the extremality of strong Markov solutions (to martingale problems). This is a joint work with David Criens.

*Speaker

Revisiting integral functionals of geometric Brownian motion

Lioudmila Vostrikova * [1]

[1] Université d'Angers – France

In this talk we revisit the integral functional of geometric Brownian motion

$$I_t = \int_0^t e^{-(\mu_s+\sigma W_s)} ds,$$

where $\mu \in \mathbb{R}$, $\sigma > 0$ and $(W_s)_{s>0}$ is a standard Brownian motion. Specifically, we calculate the Laplace transform in t of the cumulative distribution function and of the probability density function of this functional. This is joint work with Elena Boguslavskaya.

*Speaker

Convergence of Sinkhorn's Algorithm for Entropic Martingale Optimal Transport Problem

Xiaozhen Wang * [1], Fan Chen [2] , Zhenjie Ren [3] , Giovanni Conforti [4]

[1] Université Paris-Dauphine, PSL – France
[2] Shanghai Jiaotong University – China
[3] Université d'Évry Val d'Essonne, Université Paris-Saclay – France
[4] Unversity of Padova, Italy

In this paper, we study the Entropic Martingale Optimal Transport (EMOT) problem on R. We begin by introducing the dual formulation and prove the exponential convergence of Sinkhorn's algorithm on the dual potential coefficients. Our analysis does not require prior knowledge of the optimal potential and confirms that there is no primal-dual gap. Our findings provide a theoretical guarantee for solving the EMOT problem using Sinkhorn's algorithm. In applications, our result provides insight into the calibration of stochastic volatility models, as proposed by Henry-Labordere.

*Speaker

Particle methods for mixed Nash equilibria of continuous games

Guillaume Wang * [1], Lenaic Chizat [1]

[1] EPFL – Switzerland

For a two-player zero-sum game, the minimax theorem guarantees the existence of a mixed-strategy Nash equilibrium (MNE). Given oracle access to the game's payoff function, how to algorithmically identify a MNE? For games with a finite number of pure strategies, provably convergent algorithms are well-known. I will present two particle-based algorithms to tackle the case where the sets of pure strategies are continuous, e.g., Euclidean spaces, with a computational complexity that depends mildly on the dimension. The first one, Conic Particle Proximal Point, provably converges under mild non-degeneracy assumptions. The second one, Mean-Field Langevin Descent-Ascent, is stochastic in nature and provably converges to a regularized notion of MNE, but certain aspects of its convergence behavior remain to be understood. Talk based on joint work with Lénaïc Chizat.

*Speaker

Market Equilibrium under Proportional Transaction Costs in a Stochastic Factor Model

Mihail Zervos * [1]

[1] London School of Economics – United Kingdom

We consider an economy with two agents. Each of the two agents receives a random endowment flow. We model this cumulative flow as the stochastic integral of a deterministic function of the economy's state, which we model by means of a general Ito diffusion. Each of the two agents has mean-variance preferences with different risk-aversion coefficients. The two agents can also trade a risky asset. We determine the agents' optimal equilibrium trading strategies in the presence of proportional transaction costs. In particular, we derive a new free-boundary problem that provides the solution to the agents' optimal equilibrium problem. Furthermore, we derive the explicit solution to this free-boundary problem when the problem data is such that the frictionless optimiser is a strictly increasing or a strictly increasing and then strictly decreasing function of the economy's state.

*Speaker

Most exciting games-multidimensional case

Xin Zhang * [1]

[1] New York University – United States

In this talk, we will answer the question "what are the most exciting games" proposed by David Aldous. Mathematically, it can be formulated as a specific relative entropy minimization problem. We characterize the minimizer through a novel connection with the Monge Ampere equation on probability simplices.

*Speaker